

# Circle Geometry

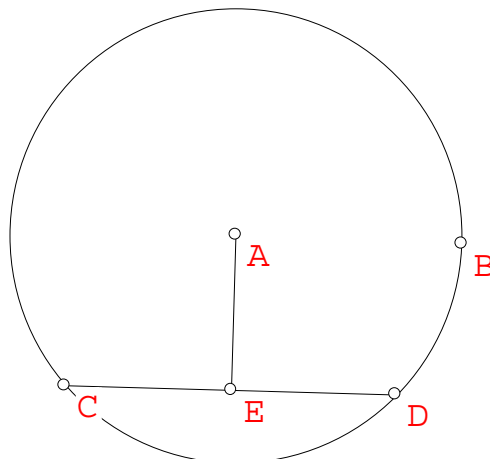
## Setup

1. Open Wingeom → Window → 2 dim then maximize your screen.
2. Go to **Point** menu and place a ✓ beside **Right Btn** plant.
3. If the toolbar is not showing then go to **Btns** → **Open toolbar**.
4. Later when using the program if the points are hidden by the letters select **Edit** → **Labels** → **Offset**. (This has only to be done once).
5. Remember at all times errors in your diagrams can be repaired by using **Edit** → **Undo**.

## Chord Properties

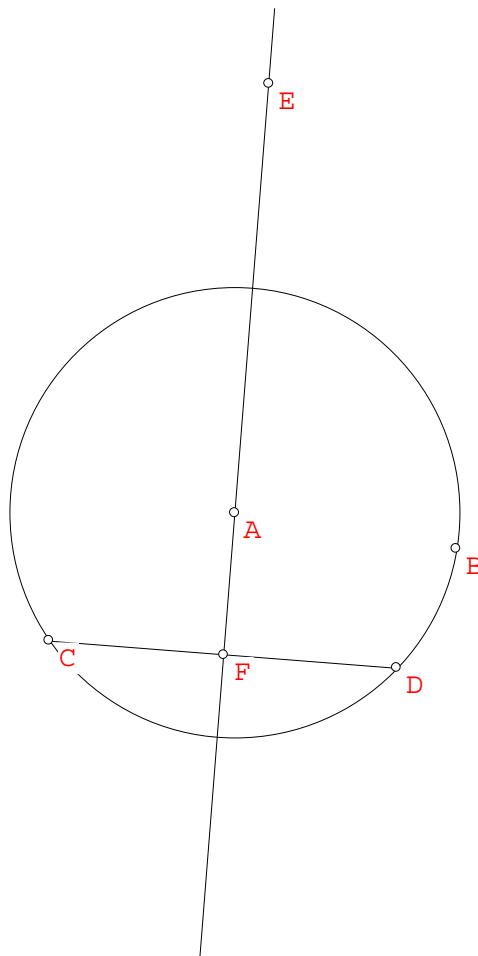
### Section A

1. On the toolbar select **circles** and right click in the centre of your screen to place point A. Then left click on A and holding the button down drag away from A to form a circle.
2. Right click to place points C and D on the circle as shown.
3. Choose **segments** on the toolbar and left click on C and drag across to D to form CD.
4. Now go to **Point** → **Midpoint** and **Mark**.
5. Left click on the new point E and drag across to join A.
6. On the menu bar select **Meas** and type in  $\angle CEA$  and Enter then  $\angle DEA$  and Enter. Always close the dialogue box after measuring. This applies to all dialogue boxes.
7. Finally on the toolbar choose **drag vertices** and left click on D and holding the button drag D around and notice the sizes of the marked angles.
8. Complete the following statement :  
The line joining the midpoint of a chord to the centre of a circle is \_\_\_\_\_ to the chord.



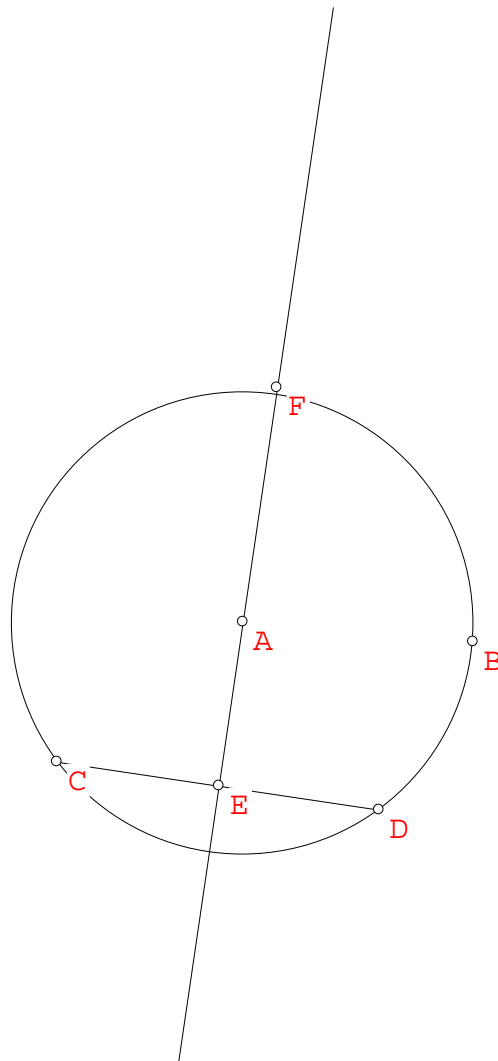
## Section B

1. Close the last window say **no** to **save changes** and go through **Set up** again as above.
2. Create a new circle and chord CD as before.
3. This time go to **Line**→**Perpendicular**→**General** and draw a line perpendicular to CD through A. Close the dialogue box.
4. Choose **Point**→**Intersection**→**Line-line** and **mark** the intersection of CD and AE.
5. Select **Meas** and type in CF and FD. Close the box.
6. Click on **drag vertices** and move B and D around.
7. Complete the statement:  
The line from the centre of a circle perpendicular to a chord \_\_\_\_\_  
the chord.



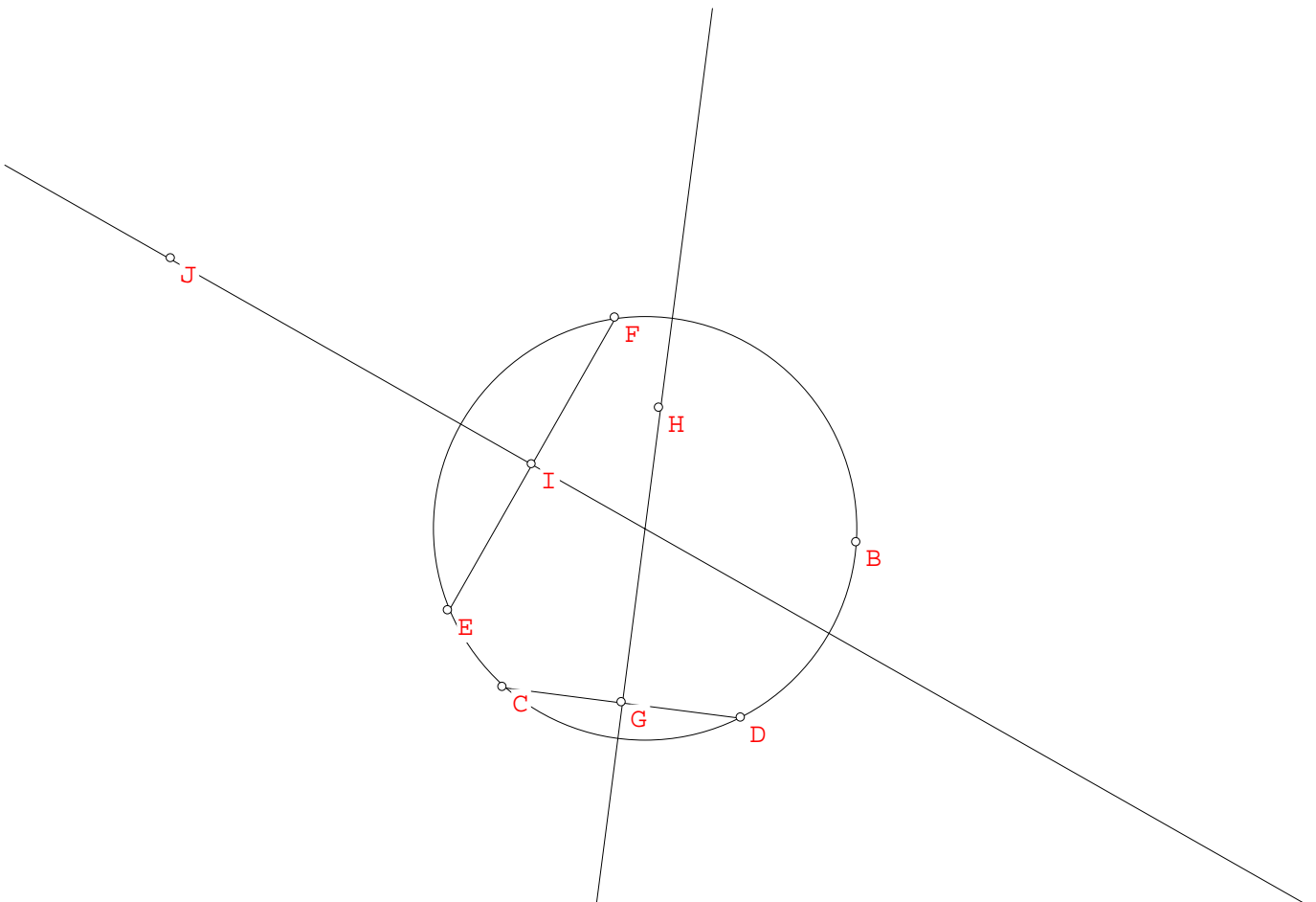
### Section C

1. Again close the last window and go through **Set up**.
2. Draw a new circle and chord CD as before.
3. Go to **Line**→**Perpendiculars**→**Perp bisector** and type CD then hit **ok**.
4. Select **drag vertices** and by moving points see if you can move EF off the centre of the circle.
5. Complete the statement:  
The perpendicular bisector of a circle passes through \_\_\_\_\_ of the circle.



## Section D

1. Save effort this time by going to **Edit**→**Undo** twice. This will take out the line EF and the point E.
2. Create an additional chord EF as well, angled to CD as shown. Click on **Point**→**Right-btn plant** to turn this feature on again.
3. Now go to **Edit**→**Labels**→**Individual** and type in A. Remove the tick in **show label** and select **no mark** and **apply**. This hides the centre A of the circle.
4. Choose **Line**→**Perpendiculars**→**Perpendicular bisectors** and type in CD, EF in the dialogue box then hit **ok**.
5. Now go top **Edit**→**Labels**→**Individuals**. Type in A and tick **show label** and select **circle** and **apply** to turn A back on.
6. Notice that the hidden centre lay on the intersection of the perpendicular bisectors of the chords.
7. Select **drag vertices** and move the ends of each chord to check this is always so.
8. This process can be used to find the centre of a metal disk if we wish to use it as a wheel.

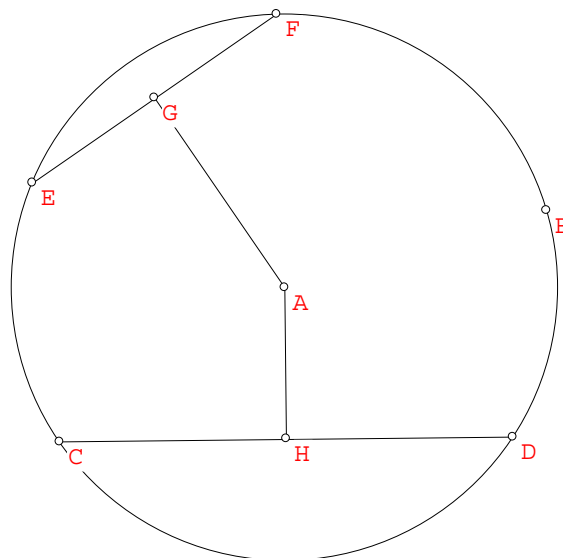


## Section E

1. Close the last window and do **Set up**.
2. Create a circle and the two chords CD and EF in similar positions as before but make EF clearly shorter than CD.
3. Go to **Point**→**Midpoints** and enter EF then **mark** then edit the dialogue box to CD and **mark**.
4. With **segments** turned on join AG and AH and close the dialogue box.
5. Choose **Edit**→**Decimals** and type in 2 in **decimal places**.
6. Select **Meas** and type in (and hit enter after each one) CD, EF, AG, AH. Close the box.
7. Now turn **drag vertices** on and move F carefully until EF is the same length as CD.
8. Notice the lengths of AG and AH.
9. Now let's reverse the process. Drag D to increase the length of CD and then watching the lengths of AH and AG move the point E until AH = AG.
10. Complete the statements:  
Chords which are equidistant from the centre of a circle are \_\_\_\_\_.

The converse statement is also true:

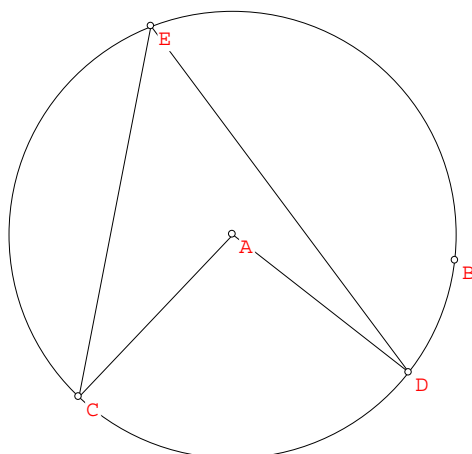
Chords which are equal are \_\_\_\_\_ from the centre of a circle.



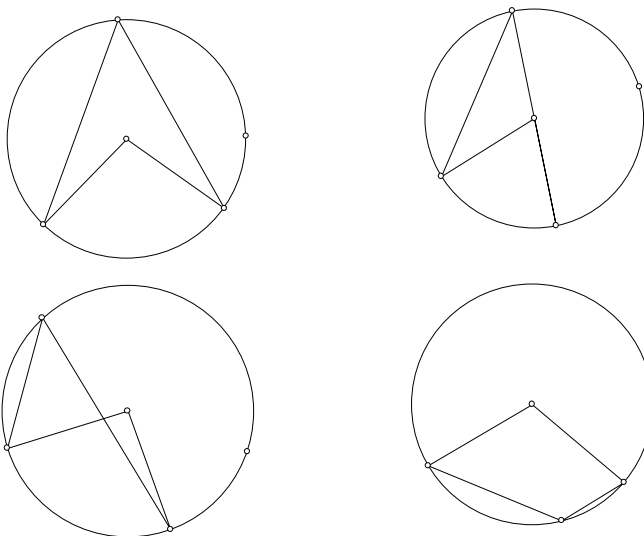
## Angle Properties

### Section A

1. Close the last window and do **Set up**.
2. Create a circle and right click to place points C,D,E as shown
3. Change toolbar to **segments** and join AC,AD,CE,ED



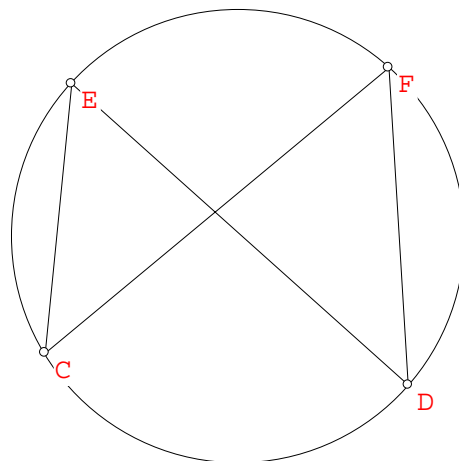
4. **Edit**→ **Decimals** to 1 place
5. Select **Meas** and type in  $\angle CED$  and enter then  $\angle CAD$  plus enter.
6. Switch **toolbar** to **drag vertices** and move D or E to investigate the relationship between these two angles. See if you can create the varieties below. It may appear that the relationship has broken down in the last diagram but can you see why it has not?



6. Finally make  $\angle CAD$  180 degrees and note that  $\angle CED = 90^\circ$ . Drag E and see that. "The angle in a semi-circle is a \_\_\_\_\_."

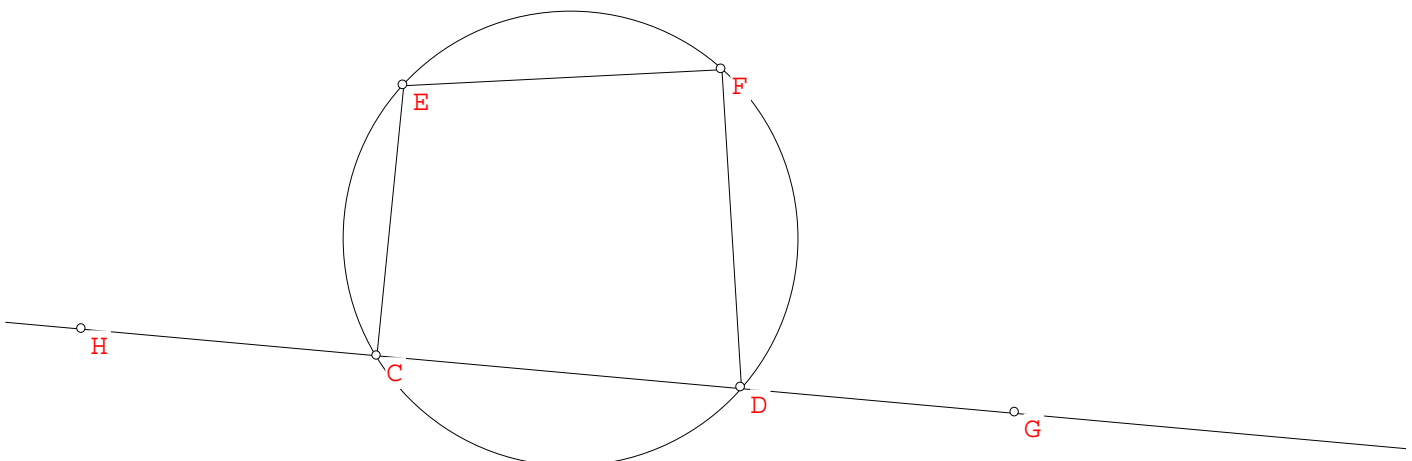
## Section B

1. Close the last exercise and create a new circle.
2. Right plant points C, D, E, F.
3. Choose **segments** and join CE, CF, DE, DF.
4. Now go to **Edit**→**Labels**→**Individual** and type in A, B. Remove the tick in **show label** and select **no mark** and **apply**.
5. **Meas** and type  $\angle CED$  then  $\angle CFD$ . Close the dialogue box.
6. Now switch to **drag vertices** and move any of the points.
7. The angles  $\angle CED$  and  $\angle CFD$  are angles standing on the same piece (arc) CD of the circle. What do you notice about the relationship between these angles?
8. Now click on **Meas** and after selecting the angle  $\angle CED$  choose **del** and repeat for  $\angle CFD$ .
9. Then type  $\angle ECF$  and next  $\angle EDF$ . Again drag the points and note the size of the angles.
10. Complete the statement:  
Angles standing on the same arc of a circle are \_\_\_\_\_.



## Section C

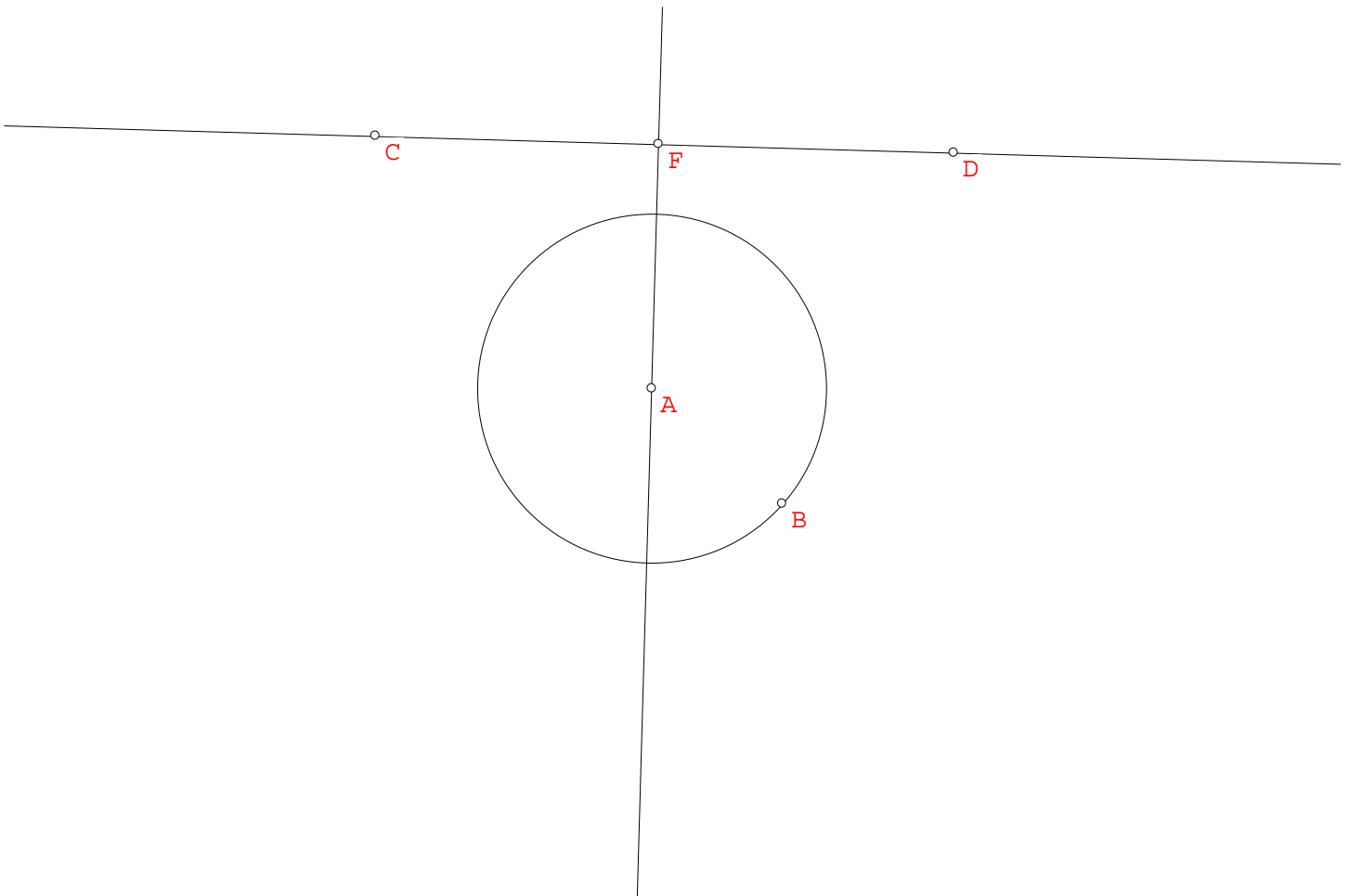
1. Instead of going through **Set up** this time go to **Edit** and select **Undo** 4 times to remove the lines from the diagram.
2. Switch to **segments** then join  $C \rightarrow E \rightarrow F \rightarrow D \rightarrow C$  to form a quadrilateral  $CEFD$ . This is known as a **CYCLIC** quadrilateral.
3. Go to **Meas** and type  $\angle CEF$  and  $\angle CDF$ .
4. **Drag vertices** and notice the changes to the angles. Can you see any relationship?
5. Now go back to **Meas** and type in  $\angle CEF + \angle CDF$ . Again move the points. Now can you see the connection? Does this apply to  $\angle ECD$  and  $\angle EFD$ ?
6. Complete the statement:  
The opposite angles of a cyclic quadrilateral are \_\_\_\_\_.
7. Now go to **Line**  $\rightarrow$  **Extensions** and type in  $CD$  and  $DC$  this will extend the line  $CD$  in both directions. Right click to place points  $G$  and  $H$  as shown (turn **Right-btn plant** on if necessary). Using the fact above, your geometry knowledge should suggest a connection between  $\angle CEF$  and  $\angle FDH$  to you.  
Can you guess what it is?
8. Now go back to **Meas** select and delete any objects present and type in  $\angle CEF$  and  $\angle FDG$ .
9. **Drag vertices** and observe the relation between the angles.
10. Can you see another pair of angles for which this connection must apply. Measure them and test your idea.
11. Complete the statement:  
The exterior angle of a cyclic quadrilateral is \_\_\_\_\_ to the interior opposite angle.



## Tangents and Circles

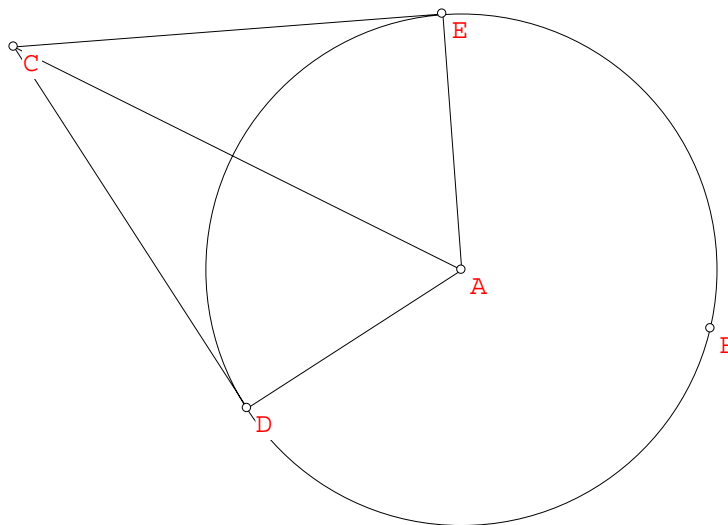
### Section A

1. Close the last window and **Set up** to draw another circle.
2. Then plant points C, D above and outside the circle as below.
3. Go to **Line**→**Lines** and type CD.
4. Then go to **Line**→**Perpendiculars**→**General** and type in CD through A and **draw**.
5. Now choose **Point**→**Intersection**→**Line-Line** and type CD and AE and **mark**.
6. Select **drag vertices** and rotate D until the line CD touches the circle at F.
7. You may find it helpful to turn F off. (**Edit** →**Labels**→**Individual** type F and deselect **show label** and choose **no mark** and **apply**).
8. Rotate D until F sits on the other side of the circle.
9. Notice that when AF becomes a radius length the line CD just touches the circle. It becomes a tangent and the radius and tangent are at right angles to each other.
10. Let's check this. First move F off the circle. Go to **Line**→**Tangent** and type in **to circle** AB and **drawn from** C and **ok**.
11. Switch on **segments** and join AG then **Meas**  $\angle$ CGA.
12. **Drag vertices** by moving C and notice that  $\angle$ CGA is fixed and that when F is on the circle CF becomes a tangent.
13. Complete the statement:  
The radius drawn to the point of contact of a tangent to a circle is \_\_\_\_\_ to the tangent.



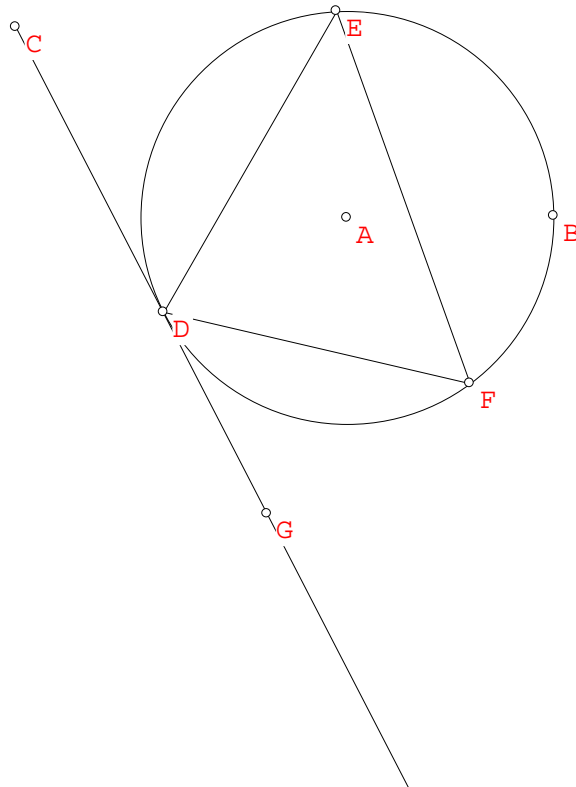
## Section B

1. Close the last window and draw a new circle.
2. Place a point C outside the circle and go to **Line**→**Tangents** and choose to circle AB from C and **ok**.
3. **Meas** CD and CE.
4. **Drag vertices** and move C.
5. Complete the Statement:  
Tangents drawn from an external point to a circle are \_\_\_\_\_.
6. Now switch to **segments** and join AD, AE, AC.
7. **Meas**  $\angle CAD$  and  $\angle CAE$  as well as  $\angle ECA$  and  $\angle DCA$ .
8. **Drag vertices** and move C and note the result.



### Section C

1. Using the last figure go to **Edit**→**Delete**→**Line** and enter AC,AD,AE,CE
2. Place a point F as shown.
3. Using **segments** join DE, DF, FE.
4. **Meas** hold down Shift while you select then turn off the current measures with **del** and type in  $\angle CDE$  and  $\angle DFE$ .
5. **Drag vertices** and move F.
6. Complete the statement:  
The angle between a tangent and a chord is \_\_\_\_\_ to the angle subtended (contained) by the chord in the alternate segment.
7. Go to **Line**→**Extensions** and type CD and **ok**.
8. Place G on the extension and **Meas**  $\angle DEF$  and  $\angle GDF$ . **Drag vertices** and move F.



## Products of Chords

### Section A

1. Close the last window and create a new circle.
2. Plant point C inside the circle above A and plant points D and E on the circle as shown.
3. Turn off A by **Edit** → **Labels** → **Individual** type A and deselect **show label** and choose **no mark** and **apply**
4. Using **segments** join CD and CE.
5. Go to **Line** → **Extensions** and type in DC, EC and **ok**.
6. Then go to **Point** → **Intersection** → **Mixed** and type in line DC and circle AB then repeat for line EC and circle AB.
7. Then using **Line** → **Extensions** type in DF, EH and **ok** to remove the parts outside the circle..
8. **Meas** the chord segment products  $EC \cdot CH$  and  $DC \cdot CF$ .
9. **Drag vertices** and move B or C inside the circle. Take note of the products of the chord segments.
10. Now drag C outside the circle opposite point B. What happens to the chord products?
11. Finally with C outside the circle move C so that F almost merges with D (note that the product becomes undefined if you merge F and D) so that CF becomes a tangent to the circle. Then  $DC = CF$  and  $EC \cdot CH = CD^2$ .

